

Behavioural Finance

Lecture 10 Out of Sequence... Modelling Endogenous Money

Circuit model of endogenous money: recap

- All exchanges 3 sided in Monetary economy
 - Seller
 - Buyer
 - Bank recording transfer of money from buyer to seller
- System driven by loans from bank sector to firm sector
- Lender
 - Must not exploit "seignorage" if system to function
 - Can & does create credit money
 - But can't "spend own notes"
- Necessarily dynamic process
 - Must be modelled using dynamic tools
 - Conventional economic tools (diagrams, partial & general equilibrium) are not dynamic...

Aside on subject content

- 2 reasons for change from investor & market behaviour
 - To financial macroeconomics
- (1) Change in teaching staff just before session
 - Subject was to have been $\frac{1}{2}$ me on behavioural economics + $\frac{1}{2}$ Craig Ellis on behavioural finance
 - Not possible when Craig seconded to work on AUQA
- (2) My perspective on implications of behavioural research on economics & finance
 - Role of agents in economic theory overstated
 - Better for economics to go "back to basics"
 - Consider dynamics of economic-financial system first
 - Later add agent behaviour as embellishment of this fundamental knowledge...

Aside on subject content

- Fundamental scientific behaviour not practised by economics in general
 - Treat economy as dynamic system
 - Build causal model of relations between its components
 - Instead, nonsense of treating economy "as if" static
 - Theory ignores feedbacks, non-equilibrium behaviour, basic issues of change, time
- To "do the basics", have to do dynamic analysis where causal relations between entities clearly stated
 - So you need to know the basics of dynamic analysis
 - But you haven't ever been taught them before...
 - So unavoidable introductory stuff in this lecture
 - But also world's first model of "the circular flow"

Aside on subject content

- This lecture
 - Explains basics of dynamic modelling
 - Shows how to simulate dynamic systems
 - Develops model of Monetary Circuit that
 - Confirms that a pure-credit economy "works"
 - Capitalists can borrow money, invest & make a profit as Keynes thought
 - Graziani "losses in the Circuit" arguments result from not understanding dynamic analysis
 - Model is world's first explicitly monetary "circular flow" model of the economy.

Dynamic modelling—an introduction

- Dynamic systems *necessarily* involve time
- Simplest expression starts with definition of the percentage rate of change of a variable:
 - "Population grows at 1% a year"
 - Percentage rate of change of a variable y is
 - Slope of function w.r.t. time (dy/dt)
 - Divided by current value of variable (y)
 - So this is mathematically $\frac{1}{y} \frac{dy}{dt} = .01$
- This can be rearranged to... $\frac{dy}{dt} = .01 \times y$
- Looks very similar to differentiation, which you have done... but essential difference: **rate of change of y is some function of value of y itself.**

Dynamic modelling—an introduction

- Dependence of rate of change of variable on its current value makes solution of equation much more difficult than solution of standard differentiation problem
- Differentiation also normally used by economists to find minima/maxima of some function
 - "Profit is maximised where the rate of change of total revenue equals the rate of change of total cost" (blah blah...)
 - Take functions for TR, TC
 - Differentiate
 - Equate
 - Easy! (also wrong, as covered in earlier lecture...)
- However differential equations...

Dynamic modelling—an introduction

- Have to be *integrated* to solve them:

$$\frac{1}{y} \frac{dy}{dt} = a \xrightarrow{\text{Rearrange}} \frac{dy}{y} = a \cdot dt$$

$$\int \frac{dy}{y} = \int a \cdot dt \xrightarrow{\text{Integrate}} \ln(y) = a \cdot t + c_0$$

$$y(t) = e^{a \cdot t + c_0} = e^{a \cdot t} \cdot e^{c_0} = C \cdot e^{a \cdot t}$$

- Constant is value of $y(0) = e^{a \cdot 0 + c_0} = e^0 \cdot e^{c_0} = C \cdot 1 = C$ at time $t=0$:

Dynamic modelling—an introduction

- Simple model like this gives
 - Exponential growth if $a > 0$
 - Exponential decay if $a < 0$
- But unlike differentiation
 - Where most functions *can* be differentiated...
 - Most functions **can't** be integrated
 - no simple solution can be found; and also
- Models can also be inter-related
 - Two variables x & y (and more: w & z & ...)
 - y can depend on itself and x
 - x can depend on itself and y
 - All variables are also functions of time**
- Models end up much more complicated...**

Dynamic modelling—an introduction

- Simple example: relationship of fish and sharks.
- In the absence of sharks, assume fish population grows smoothly:
 - "The rate of growth of the fish population is $a\%$ p.a."

$$\frac{1}{F} \frac{dF}{dt} = a \xrightarrow{\text{Rearrange}} \frac{dF}{F} = a \cdot dt$$

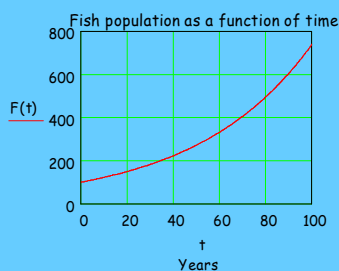
$$\int \frac{dF}{F} = \int a \cdot dt \xrightarrow{\text{Integrate}} \ln(F) = a \cdot t + c_0$$

$$F(t) = e^{a \cdot t + c_0} = e^{a \cdot t} \cdot e^{c_0} = C \cdot e^{a \cdot t}$$

Dynamic modelling—an introduction

- Simulating gives exponential growth if $a > 0$

$$a := .02 \quad F_0 := 100 \quad F(t) := F_0 \cdot e^{a \cdot t}$$



Dynamic modelling—an introduction

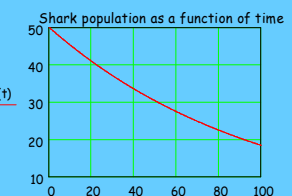
- Same thing can be done for sharks in the absence of fish:
 - Rate of growth of shark population equals $c\%$ p.a.
 - But here c is negative $S(t) = C_2 \cdot e^{c \cdot t}$

- But we know fish and sharks interact:

- "The rate of change of fish populations is also some (negative) function of how many Sharks there are" $\frac{1}{F} \cdot \frac{dF}{dt} = a - b \cdot S$

- "The rate of change of shark population is also some (positive) function of how many Fish there are" $\frac{1}{S} \cdot \frac{dS}{dt} = c + d \cdot F$

$$c := -.01 \quad S_0 := 50 \quad S(t) := S_0 \cdot e^{c \cdot t}$$



Dynamic modelling—an introduction

- Now we have a model where the rate of change of each variable (fish and sharks) depends on its own value **and** the value of the other variable (sharks and fish):

$$\frac{1}{F} \cdot \frac{dF}{dt} = a - b \cdot S \implies \frac{dF}{dt} = a \cdot F - b \cdot S \cdot F$$

$$\frac{1}{S} \cdot \frac{dS}{dt} = c + d \cdot F \implies \frac{dS}{dt} = c \cdot S + d \cdot F \cdot S$$

- This can still be solved, with more effort (don't worry about the maths of this!):

Dynamic modelling—an introduction

- But for technical reasons, this is the *last level of complexity that can be solved*

$$d \ln(F) = a - b \times S \times dt$$

$$\int d \ln(F) = \int a - b \times S \times dt$$

$$\ln(F) = (a - b \times S) \times t + c$$

$$F = C_1 \times e^{(a - b \times S(t)) \times t}$$

Similarly

$$S = C_2 \times e^{(-c + d \times F(t)) \times t}$$

- Add an additional (nonlinearly related) variable—say, seagrass levels—and **model cannot be solved**

But there are other ways...

- Mathematicians have shown that unstable processes can be simulated
- Engineers have built tools for simulating dynamic processes...

Dynamic modelling—an introduction

- (1) Enter (preferably empirically derived!) parameters:

Parameters

- $a := 1$ Annual population growth rate of fish in absence of sharks
- $b := .1$ Impact of each shark on fish population growth rate
- $c := 1$ Death rate of sharks in absence of fish
- $d := .0001$ Impact of each fish on survival of sharks

- (2) Calculate equilibrium values: **Simulate!**

- (3) Rather than stopping there: **Simulate!**
- Given $\frac{d}{dt} F(t) = F(t) \cdot (a - b \cdot S(t))$ $F(0) = 8000$ Sharks $S_e = \frac{c}{b}$ $S_e = 10$

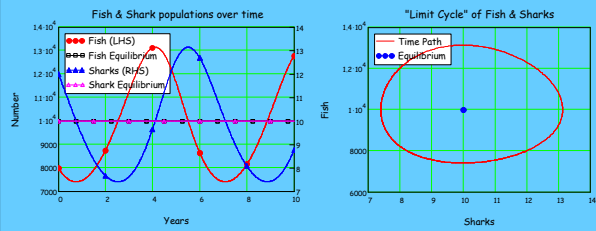
$$\frac{d}{dt} S(t) = S(t) \cdot (-c + d \cdot F(t))$$

$$\begin{bmatrix} \dot{F} \\ \dot{S} \end{bmatrix} = \text{Odesolve} \left[\begin{bmatrix} F \\ S \end{bmatrix}, t, 10, 1000 \right]$$

Graph the results!

Dynamic modelling—an introduction

- System is never in equilibrium
 - Equilibrium "unstable"—repels as much as it attracts
- Commonplace in dynamic systems
 - Systems are always "far from equilibrium"
 - (What odds that the actual economy is in equilibrium?...)



Dynamic modelling—an introduction

- That's the "hard" way; now for the "easy" way...
- Differential equations can be simulated using flowcharts
 - The basic idea...
 - Numerically integrate the rate of change of a function to work out its current value
 - Tie together numerous variables for a dynamic system
 - Consider simple population growth:
 - "Population grows at 2% per annum"

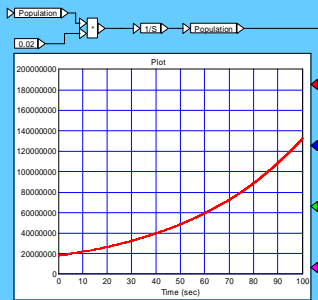
Dynamic modelling—an introduction

- Representing this as mathematics, we get $\frac{1}{P} \frac{dP}{dt} = .02$
- Next stage of a symbolic solution is $\frac{dP}{dt} = .02 \cdot P$

- Symbolically you would continue, putting "dt" on the RHS; but instead, numerically, you **integrate**: $P = \int .02 \cdot P \cdot dt$

- As a flowchart, you get:
- Read it backwards, and it's the same equation:
- Feed in an initial value (say, 18 million) and we can simulate it (over, say, 100 years)

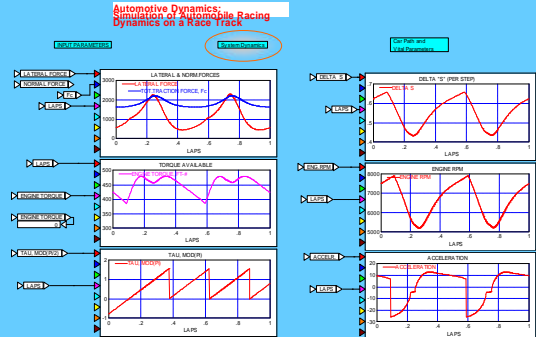
Dynamic modelling—an introduction



- MUCH more complicated models than this can be built...

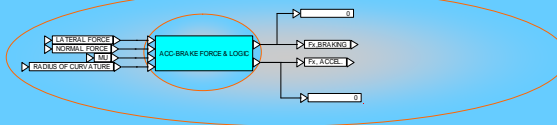
Dynamic modelling—an introduction

- Models can have multiple interacting variables, multiple layers...; for example, a racing car simulation:



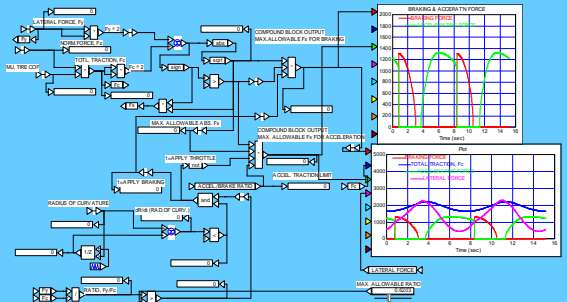
Dynamic modelling—an introduction

- "System dynamics" block has these components:



- And this block has the following components...

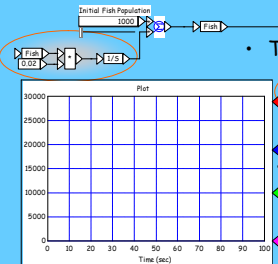
Dynamic modelling—an introduction



- **This is not toy software:** engineers use this technology to design actual cars, planes, rockets, power stations, electric circuits...

Dynamic modelling—an introduction

- Let's use it to build the Fish/Shark model
 - Start with population model, only
 - Change Population to Fish
 - Alter design to allow different initial numbers

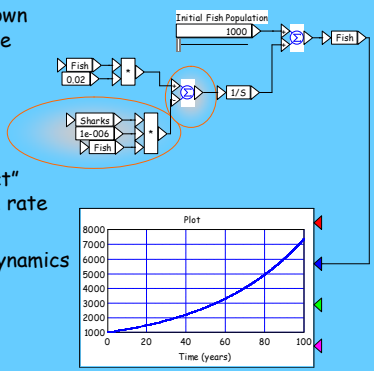


This is equivalent to first half of
$$\frac{dF}{dt} = a \cdot F - b \cdot S \cdot F$$

To add second half, have to alter part of model to LHS of integrator

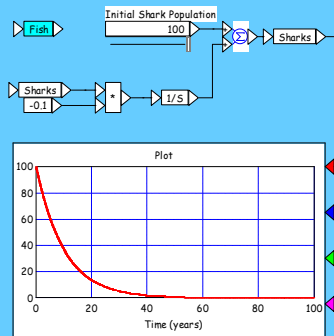
Dynamic modelling—an introduction

- Sharks just shown as constant here
- Sharks "subtract" from fish growth rate
- Now add shark dynamics



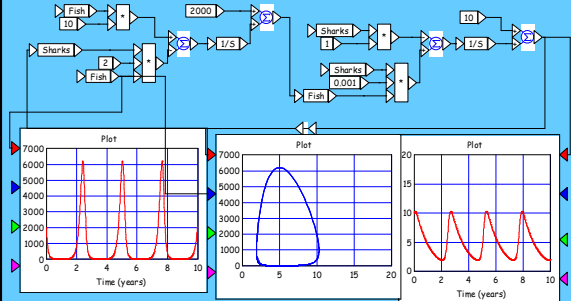
Dynamic modelling—an introduction

- Shark population declines exponentially, just as fish population rises
 - (numbers obviously unrealistic)
- Now add interaction between two species



Dynamic modelling—an introduction

- Model now gives same cycles as seen in mathematical simulation.
 - Now to apply this to endogenous money!



But first another approach to dynamics...

- Flowchart modelling fabulous for simulation
 - But difficult for economists to understand
- Another approach I've developed is much more natural for accounting for financial flows:
 - Use "Double Entry Book-Keeping" to record flows
 - The idea
 - Each column is an account
 - Rows record transactions between accounts
 - Add up columns, and you get dynamic model...
 - Approach still in its infancy
 - But much easier to follow than flowcharts...

"Double entry accounting" as a dynamic system...

- Each column represents a particular stock
- Each row entry represents a flow between stocks
- Specify relations between system states across rows...

Dynamic System					
"System States"					
	Stock A	Stock B	Relations	Stock Z	Accounting
Flows	Flow 1	- Flow 1	→	- Flow 2	Sum (=0)
	Sum
	$\frac{d}{dt} A(t)$	$\frac{d}{dt} B(t)$		$\frac{d}{dt} Z(t)$	

- To generate the model, add up each column
 - Sum of column is "differential equation" for stock
 - Continuous time, not "discrete" time
 - Strictly monetary model developed...

Basic Circuitist Dilemmas solved

- Initial Circuitist Model (Graziani pp. 4-6)
- "The first step in the economic process is the decision taken by banks of granting credit to firms in order to enable them to start production..."
- Bank doesn't need reserves from which to lend
 - Bank status allows it to create money "from nothing"
 - Book-keeping action of
 - Making entry of \$L in deposit account
 - And simultaneous \$L entry in debt account
 - Creates loan and money simultaneously...
 - "The first step in the economic process is the decision taken by banks of granting credit to firms in order to enable them to start production" (4)

Basic Circuitist Dilemmas solved

- So system starts with Bank Sector making loan of \$L to Firm Sector
 - Two accounts needed:
 - Money added to Deposit Account F_D
 - Record of Debt kept in Loan Account F_L
 - Amount of \$L entered into both initially
 - Use double entry-table to record flows initiated by loan
 - Compound interest on loan
 - Payment of interest on deposit balance
 - All money transfers begin & end in bank accounts...

Basic Circuitist Dilemmas solved

Loan contract gives bank right to compound debt at rate of interest on loans
Interest payment from bank to firm
Interest payment from firm to bank
Bank records payment of interest as reduction in outstanding debt

Type of Account (Bank point of view)	Asset	Liability	Income
Name	Firm Loan	Firm Deposit	Bank Deposit
Symbol	F_L	F_D	B_D
Flows between accounts			
Compound Interest	A		
Deposit Interest		+B	-B
Pay Interest	-C	-C	+C
Sum of flows	A-C	B-C	C-B

Basic Circuitist Dilemmas solved

- Firm pays interest to Bank at loan rate r_L ;
- Bank pays interest to Firm at deposit rate r_D ;
- So **A** is interest rate on debt (r_L) times debt F_L ;
- B** is interest rate on deposits (r_D) times deposit F_D ;
- If firm pay all interest on debt, **C** is the same as **A** - (and debt F_L therefore remains constant)

Type of Account	Asset	Liability	Income
Name	Firm Loan	Firm Deposit	Bank Deposit
Symbol	F_L	F_D	B_D
Compound Interest	$A=r_L \cdot F_L$		
Deposit Interest		$+B=r_D \cdot F_D$	-B
Pay Interest	$-C=r_L \cdot F_L$	-C	+C
Sum of flows	0	$r_D \cdot F_D - r_L \cdot F_L$	$r_L \cdot F_L - r_D \cdot F_D$

Basic Circuitist Dilemmas solved

- So our basic model so far is that:
 - Rate of change of firm sector's debt = zero
 - Rate of change of firm sector's deposit account is interest payments on deposit minus interest payments on loan
 - $r_D \cdot F_D - r_L \cdot F_L$
 - Rate of change of bank sector's income account is the opposite:
 - $r_L \cdot F_L - r_D \cdot F_D$
- Much more needed, but we can model this already...
 - Firstly, the system I use to create the model
 - Written in Mathcad, but reproducible in any modern mathematics program (Mathematica, Maple, etc.) with symbolic & numeric processing routines:

Basic Circuitist Dilemmas solved

- Basic system is:

$$S_1 := \begin{pmatrix} \text{"Type"} & 1 & -1 & 0 \\ \text{"Name"} & \text{"Firm Loan"} & \text{"Firm Deposit"} & \text{"Bank Deposit"} \\ \text{"Symbol"} & F_L(t) & F_D(t) & B_D(t) \\ \text{"Compound Interest"} & A & 0 & 0 \\ \text{"Deposit Interest"} & 0 & B & -B \\ \text{"Pay Loan Interest"} & -C & -C & C \end{pmatrix}$$
- Make substitutions for **A**, **B** & **C**:
 - $A := r_L \cdot F_L(t)$
 - $B := r_D \cdot F_D(t)$
 - $C := r_L \cdot F_L(t)$
- And program returns:
 - Enter reasonable values:
 - $r_D = 1\%$, $r_L = 5\%$
 - And initial conditions:
 - Initial Loan = \$100
 - Initial Deposit = Loan
 - Basic principle of endogenous money creation...
- Simulate...

Basic Circuitist Dilemmas solved

- Numbers show predictable result
 - firms go broke!**...
- Loan remains constant
 - No repayment (yet)
- Firm Deposit Account Falls
 - (Since $r_L \cdot F_L > r_D \cdot F_D$ at start)
 - (And F_D falls while F_L remains constant)
- Bank Deposit Account Rises
 - (Since $r_L \cdot F_L > r_D \cdot F_D$ at start)
 - (And F_D falls while F_L remains constant)

- But what if we add in next stage of Circuit model?:
 - Firms hire workers
 - Worker and banks consume output of factories...
- Is the system viable???

Basic Circuitist Dilemmas solved

- Need one more account for this:
 - 2 new activities
 - "Household Deposit" H_D
 - Workers paid wages
 - Table now has 4 columns:
 - Consumption

Type of Account	Asset	Liability (Deposits by non-bank public)		Income
Name	Firm Loan	Firms	Households	Bank
Symbol	F_L	F_D	H_D	B_D
Compound Interest	A			
Deposit Interest		+B		-B
Pay Interest	-A	-A		+A
Pay Wages			+E	
Interest on HH			-E	
Consume		E+F	-E	-F
Sum of flows	0	$B+E+F-(A+C)$	$C+D-E$	$A-(B+D+F)$

Basic Circuitist Dilemmas solved

- Bank accounts will stabilise if flows in equal flows out:

Type of Account	Asset	Liability		Income
Name	Firm Loan	Firms	Households	Bank
Symbol	F_L	F_D	H_D	B_D
Compound Interest	A			
Deposit Interest		+B		-B
Pay Interest	$-C(-A)$	-C		+C
Pay Wages		-D	+D	
HH Interest			+E	-E
Consume		F+G	-F	-G
Sum of flows	0	B+F+G-(C+D)	D+E-F	C-(B+E+G)

- Loans stable since repayments=compounding
- Firm deposit stable if $B+F+G=C+D$
 - Deposit Interest + Sales = Loan Interest + Wages
- Household deposit stable if $D+E=F$
 - Worker Consumption = Wages + Interest on HH Deposits
- Bank deposit stable if $C=B+E+G$
 - Loan interest = Deposit Interest + Banker Consumption

Basic Circuitist Dilemmas solved

- Doesn't sound too difficult
 - unlike Graziani's arguments that constant economic activity requires rising debt levels to pay interest...

- New system is:
- New parameters

- w : rate of flow of funds from firms to workers as wages for working in factories
- ω : consumption rate of workers
- β : consumption rate of bankers

Type	1	-1	-1	0
Name	Firm Loan	Firm Deposit	Household Deposit	Bank Deposit
Symbol	$F_L(t)$	$F_D(t)$	$H_D(t)$	$B_D(t)$
Compound Interest	A	0	0	0
Deposit Interest	0	B	0	-B
Pay Loan Interest	-C	-C	0	C
Pay Wages	0	-D	D	0
Household Interest	0	0	E	-E
Consumption	0	F+G	-F	-G

Related to balances in accounts
 $D = w F_D(t) - E = \tau_D H_D(t) - \tau_B B_D(t)$

- System equations are...

$$\text{SystemODEs}(S) \rightarrow \begin{cases} \frac{dF_L(t)}{dt} = 0 \\ \frac{dF_D(t)}{dt} = \beta B_D(t) + \omega H_D(t) + \tau_D F_D(t) - \tau_L F_L(t) - w F_D(t) \\ \frac{dH_D(t)}{dt} = \tau_D H_D(t) - \omega H_D(t) + w F_D(t) \\ \frac{dB_D(t)}{dt} = \tau_L F_L(t) - \tau_D F_D(t) - \beta B_D(t) - \tau_B H_D(t) \end{cases}$$

Basic Circuitist Dilemmas solved

- Simulating this with trial values:

- $w=3$; $\omega=26$; $\beta=1$
- What do we get?

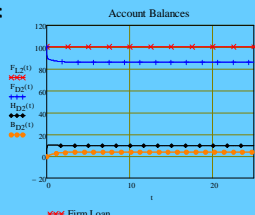
Parameters $\Delta t = 5\%$, $\Delta t = 1\%$, $\Delta t = 25\%$, $\Delta t = 100\%$, $w = 3$, $\omega = 26$, $\beta = 1$

Given $\frac{dF_L(t)}{dt} = 0$, $F_L(0) = 100$

$\frac{dF_D(t)}{dt} = \beta B_D(t) + \omega H_D(t) + \tau_D F_D(t) - \tau_L F_L(t) - w F_D(t)$, $F_D(0) = 0$

$\frac{dH_D(t)}{dt} = \tau_D H_D(t) - \omega H_D(t) + w F_D(t)$, $H_D(0) = 0$

$\frac{dB_D(t)}{dt} = \tau_L F_L(t) - \tau_D F_D(t) - \beta B_D(t) - \tau_B H_D(t)$, $B_D(0) = 0$



- Simulation results:
- All accounts stabilise at positive values...
- How do we interpret this?...

 - Can we derive incomes from account figures?
 - Are they compatible with positive incomes for all?

Basic Circuitist Dilemmas solved

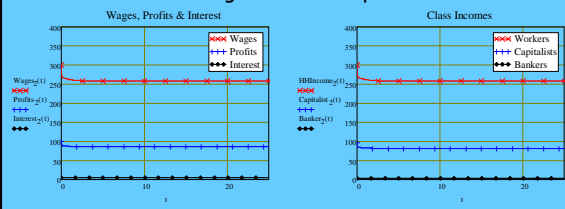
- Gross Incomes
 - Bank obviously $r_L F_L = 5\%$ of $\$100 = \5 p.a.
 - Wages obviously $w F_D = 3 \times \$86.029 = \258.088 p.a.
 - Notice annual wages much larger than initial loan
 - $\$L = \100
 - But what are profits???
 - Clue given by fact that wages much larger than initial loan
 - Money "turns over" several times in a year
 - Turnover period (time from spending M on production & getting M+ back in sales) one part
 - Wages represent workers share in surplus of outputs over inputs
 - Share of surplus going to capitalists other part

Basic Circuitist Dilemmas solved

- Call capitalist share of surplus s
 - Then workers get $(1-s)$
- Call turnover period τ_S
 - Fraction of a year that it takes to go from M to M+
 - Time between initial outlay (hire workers, pay wages) & receiving money from sale of output
- So $w = (1-s)/\tau_S$
 - And wages are $((1-s)/\tau_S) F_D$
- Profits are $(s/\tau_S) F_D$
- So given $w=3$, one possibility is
 - Capitalists share of surplus from production = 25%
 - Turnover period from M to M+ is 3 months = $\frac{1}{4}$ year
 - Profits = $0.25/\frac{1}{4} F_D = 1.0 \times \$86.029 = \$86.029$ p.a.

Basic Circuitist Dilemmas solved

- So Loan of $\$100$ generates
 - Equilibrium annual income of $F_D/\tau_S = \$344.117$
 - Which is split
 - 75% to workers = $\$258.088$
 - 25% to capitalists = $\$86.029$
 - Banks' gross interest ($\$5$) a cut from this
 - Both net and gross incomes positive:



Basic Circuitist Dilemmas solved

- What about paying back debt?
 - Debt account a record of amount you owe to bank
 - Can be reduced by repaying debt
 - But isn't "negative money"
- So another account needed to record debt repayment
 - Bank Reserves (B_R)
 - Repayment goes here
 - Seignorage if went into B_D account
 - Banks spending money they created
 - Bank reduces record of outstanding debt
- 2 operations in debt repayment
 - Transfer of money *from* Firm Deposit *to* Bank Reserve
 - Recording of repayment on Firm Loan record.

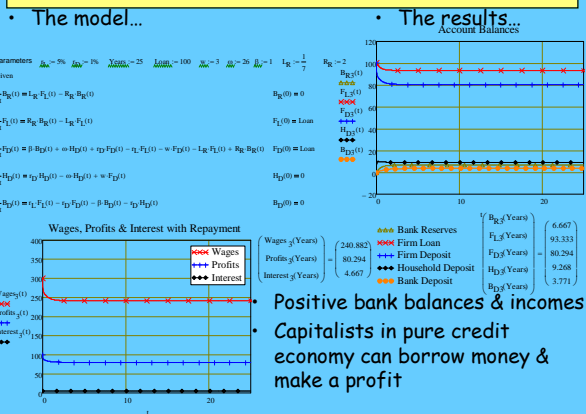
Basic Circuitist Dilemmas solved

- Bank can also lend from its Reserves, so 2 more rows:

Type of Account	Asset		Liability		Income
	Bank Reserve	Firm Loan	Firms	Households	Bank
Name	B_R	F_L	F_D	H_D	B_D
Symbol	B_R	F_L	F_D	H_D	B_D
Compound Interest		A			
Deposit Interest			+B		-B
Pay Interest		$-C(-A)$	-C		+C
Pay Wages			-D	+D	
HH Interest				+E	-E
Consume			F+G	-F	-G
Repay Debt	+H	-H	-H		
Relend Reserves	-I	+I	+I		
Sum of flows	H-I	I-H	...+I-H

- System can be stable if
 - Repayment rate=Relending rate ($H=I$)
- Simulating this...

Basic Circuitist Dilemmas solved



Basic Circuitist Dilemmas solved

- Loan of \$L causes much more than \$L turnover per year
- Profits and Wages earned from flows initiated by loan
 - Easily exceed Loan itself
 - \$321 annual incomes from \$100 loan
 - So payment of interest easy
 - Just \$4.67 gross from profits of \$80.30
 - Repayment also easy
 - And a discovery: not only "Loans create Deposits"
 - But "Loan Repayment creates Reserves"
 - Reserves stabilise at \$6.67 from zero start
- A pure credit economy "works"
 - No *necessity* for a financial crisis
 - *But* they do occur—can we work out why?

Next week

- Extending model to include production
- Explaining values of parameters
- Working out why lenders like lending too much money
- Expanding model to include growth
- Modelling a "credit crunch"
- Introducing the Financial Instability Hypothesis