Behavioural Finance

Lecture 05
Fractal Finance Markets

Recap

- Last week
  - Data strongly contradicts Capital Assets Pricing Model
    - Early apparent success a quirk
      - Short data series analysed by Fama etc.
      - Coincided with uncharacteristic market stability
  - Market highly volatile
    - Follows "Power Law" process
    - Any size movement in market possible

Overview

- Market predominantly not random
- But pattern of market movements very hard to work out
- Fractal markets hypothesis
  - Market dynamics follow highly volatile patterns

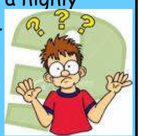
The dilemma

- CAPM explained difficulty of profiting from patterns in market prices
  - Via "Technical Analysis" etc.
- On absence of any pattern in market prices
  - Fully informed rational traders
  - Market prices reflect all available information
  - Prices therefore move randomly
- Failure of CAPM
  - Prices don't behave like random process
- Implies there is a pattern to stock prices
  - Question: if so, why is it still difficult to profit from market price information
  - Answer: Fractal Markets Hypothesis…

Fractals

- What's a fractal???

- A self-similar pattern in data generated by a highly nonlinear process…



- Remember irrational numbers?
  - Solution to question "is the square root of 2 rational?"
    - Equal to ratio of two integers?
      - No!
- Fractals similar:
  - Can we describe landscapes using standard solids?
    - Solid cubes, rectangles, etc?

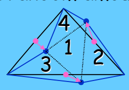
Fractals

- Does Mount Everest look like a triangle?

- Yes and No
  - Not like a single pure triangle
  - But maybe lots of irregular triangles put together…
  - Mandelbrot invented concept of "fractals" to express this
- Real-world geography doesn't look like standard solid objects from Euclidean Geometry
  - Squares, circles, triangles…
- But can simulate real-world objects by assembling lots of Euclidean objects at varying scales…

Fractals

- For example, simulate a mountain by manipulating a triangle:
  - Start with simple triangle
    - Choose midpoints of three sides
    - Move them up or down a random amount
    - Create 4 new triangles;



- Repeat
- Resulting pattern does look like a mountain…

Fractals

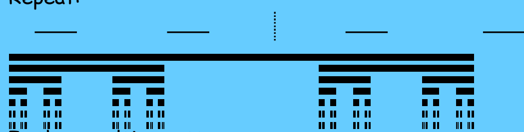
- Mandelbrot (who developed the term) then asked "How many dimensions does a mountain have?"
  - All "Euclidean" objects have integer dimensions:
    - A line: 1 dimension
    - A square: 2 dimensions
    - A sphere: 3 dimensions
- Is a picture of a mountain 2 dimensional?
  - Maybe; but to generate a 2D picture, need triangles of varying sizes
    - If use triangles all of same size, object doesn't look like a mountain
- So maybe a 2D photo of a mountain is somewhere between 1 dimension and 2?

Fractals

- A single point has dimension zero (0):
- A straight line has dimension 1:
- A rectangle has dimension 2:
- How to work out "sensible" dimension for irregular object like a mountain?
  - Consider a stylised example: the Cantor set…

Fractals

- Take a line:
- Remove middle third:
- Repeat:



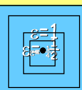
- Is the resulting pattern…
  - 1 dimensional (like a solid line);
  - 0 dimensional (like isolated points);
  - Or somewhere in between?
- A (relatively) simple measure: "box-counting" dimension…

Fractals

- How many boxes of a given size does it take to cover the object completely?
- Define box count so that Euclidean objects (point, line, square) have integer dimensions
- Dimension of something like Cantor Set will then be fractional: somewhere between 0 and 1
- Box-counting dimension a function of
  - Number of boxes needed N
  - Size of each box ε as smaller and smaller boxes used
- Measure is limit as size of box ε goes to zero of $\frac{\ln(N)}{\ln\left(1/\varepsilon\right)}$
- Apply this to an isolated point:
  - Number of boxes needed—1, no matter how small
  - $1/\varepsilon$ goes to infinity as box gets smaller

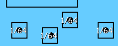
Fractals

- Single point:

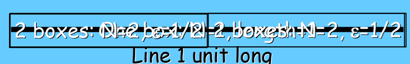


$$\lim_{\varepsilon\to 0}\frac{\ln(N)}{\ln\left(1/\varepsilon\right)}=\lim_{\varepsilon\to 0}\frac{\ln(1)}{\ln\left(1/\varepsilon\right)}=\lim_{\varepsilon\to 0}\frac{0}{\ln\left(1/\varepsilon\right)}=0$$

- Many points:



- Same result:
  - Ln(N) equals number of points (here N=4; ln(4)=0.7)
  - here $\varepsilon=1/64$; $\ln(1/\varepsilon)=4.2$; tends to infinity as $\varepsilon\to 0$
  - Any number divided by infinity is zero…
- Works for a line too:



- N function of length of boxes: $N=1/\varepsilon$
- Dimension of line is 1 as required: $\lim_{\varepsilon\to 0}\frac{\ln(N)}{\ln\left(1/\varepsilon\right)}=\lim_{\varepsilon\to 0}\frac{\ln\left(1/\varepsilon\right)}{\ln\left(1/\varepsilon\right)}=1$

Fractals

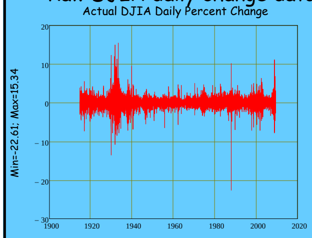
- What about Cantor set? Line 1 unit long
  - One box: N=1, length=1
- Remove middle third:
  - 2 boxes: N=2, ε=1/3
  - 2 boxes: N=2, ε=1/3
- Repeat:
  - 4 boxes
  - N=4=2^2
  - ε=1/9
  - ε=$(1/3)^2$
- Formula for each line is:
  - Number of boxes (N) equals 2 raised to power of level
    - Zeroth stage 2^0=1; 1st 2^1=2 boxes; 2nd stage 2^2=4…
  - Length of box = (1/3) raised to power of level
    - Zeroth $(1/3)^0$=1; 1st $(1/3)^1$=1/3; 2nd $(1/3)^2$=1/9
- Dimension of Cantor set = $\lim_{\varepsilon\to 0}\frac{\ln(N)}{\ln\left(\frac{1}{\varepsilon}\right)} = \lim_{\varepsilon\to 0}\frac{\ln(2^n)}{\ln\left(\frac{1}{3}^n\right)} = \frac{\ln(2)}{\ln(3)} \approx 0.63$

Fractals

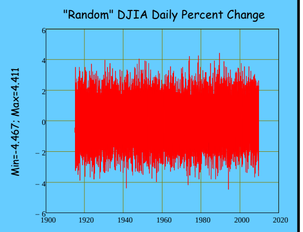
- So what's this got to do with Stock Markets?
- Basic idea behind fractals is measuring roughness
  - See Mandelbrot's lecture at MIT on this
- Euclidean objects (points, lines, rectangles, spheres) are "smooth"
  - Slope changes gradually, everywhere differentiable
  - Have integer dimensions
- Real objects are rough
  - Slope changes abruptly, everywhere discontinuous
  - Have fractal dimensions
- Stock Exchange data has "fractal" rather than "integer" dimensions, just like mountains, Cantor Set, river flows…
- Let's check it out:

Fractal Markets

- Raw DJIA daily change data is:



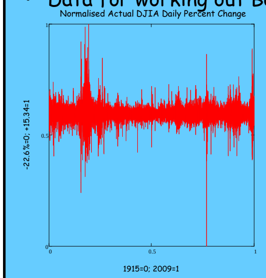
- Pseudo-random data is:

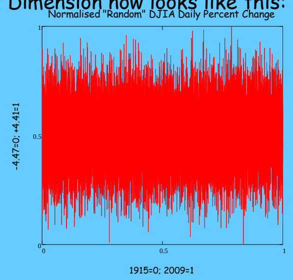


- Differences pretty obvious anyway!
  - But let's derive Box-Counting Dimension of both…
- First step, normalise to a 1 by 1 box in both directions:

Fractal Markets

- Data for working out Box Dimension now looks like this:

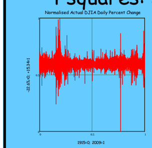


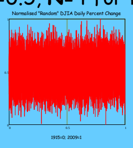


- Now start dividing graph into boxes
- and count how many squares have data in them:

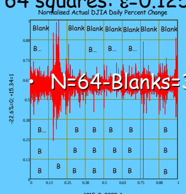
Fractal Markets

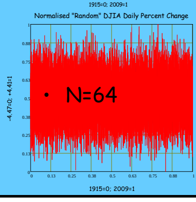
- 4 squares: ε=0.5, N=4 for both



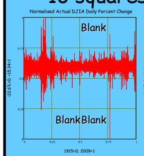


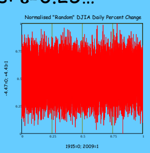
- 64 squares: ε=0.125…





- 16 squares: ε=0.25…





- N=13
- N=16

Fractal Markets

- Now write program to do automatically what we saw:
  - Break data into 8 by 8 squares
  - Work out that 34 of them have data in them…
  - Repeat for larger number of squares…

```
count ← rows(x)
ε ← matrix(steps, 1, f)
Squares ← matrix(steps, 1, f)
N ← matrix(steps, 1, f)
ε_0 ← 1
Squares_0 ← 1
N_0 ← 1
for i ∈ 1..steps − 1
    ε_i ← ε_(i−1) ÷ 2
    trace("Eta is {0}", ε_i)
    Squares_i ← 1 ÷ (ε_i)^2
    for j ∈ 0..1/ε_i − 1
        start ← j·ε_i
        end ← [(j + 1)·ε_i]
        find_min ← start
        find_max ← end
        stack_j ← sort(submatrix(x, floor(start·count), ceil(end·count) − 1, 1, 1))
        for k ∈ 0..1/ε_i − 1
            find_min ← ε_i·k
            find_max ← ε_i·(k + 1)
            start_data ← (stack_j)_0
            end_data ← (stack_j)_last(stack_j)
            N ← N + 1 if start_data ≤ find_min ≤ end_data ∨ start_data ≤ find_max ≤ end_data
    N_i ← N
    N ← 0
return augment(ε, Squares, N)
```

$DJIA_{Dim}$ =

	0	1	2
0	1	1	1
1	0.5	4	4
2	0.25	16	13
3	0.125	64	34 Correct!
4	0.0625	256	108
5	0.0313	1024	317
6	0.0156	4096	986
7	0.0078	16384	3150
8	0.0039	65536	10202
9	0.002	262144	33641

Fractal Markets

- Then apply box dimension rule: $Dimension = \lim_{\varepsilon \to 0} \frac{\ln(N)}{\ln\left(\frac{1}{\varepsilon}\right)}$

$\frac{\ln(N)}{\ln\left(\frac{1}{\varepsilon}\right)} =$

	0
0	0
1	2
2	1.85
3	1.696
4	1.689
5	1.662
6	1.658
7	1.66
8	1.665
9	1.671

- So fractal dimension of DJIA is roughly 1.67
- What about random data?

$DJIARand_{Dim} =$

	0	1	2
0	1	1	1
1	0.5	4	4
2	0.25	16	16
3	0.125	64	64
4	0.0625	256	230
5	0.0313	1024	859
6	0.0156	4096	3137
7	0.0078	16384	11620
8	0.0039	65536	42252
9	0.002	262144	151153

$\frac{\ln(N_{Rand})}{\ln\left(\frac{1}{\varepsilon_{Rand}}\right)} =$

	0
0	0
1	2
2	2
3	2
4	1.961
5	1.949
6	1.936
7	1.929
8	1.921
9	1.912

Why does this matter???

Fractals and Structure

- Truly random process has no structure
  - Say 1st 3 tosses of coin = "Heads"
  - Even though odds of 4 Heads in row very small (6/100)
  - Odds next toss = "Heads" still ½
  - Past history of tosses gives no information about next
- Fractal process has structure
  - Some dynamic process explains much of movement
    - But not all!
      - Some truly random stuff as well in data
  - But…
    - Process may be impossible to work out;
    - May involve interactions with other systems; and
    - Even if can work it out, difficult to predict

Fractals and Structure

- An example: Logistic equation $L_{t+1} = (1+a)\cdot L_t - b\cdot L_t^2$
- Developed to explain dynamics of animal populations
  - Some "prey" animals (e.g. Lemmings, Red Crabs)
    - Give birth on same day every (Lunar!) year
    - Huge numbers born relative to population
  - Survival tactic
    - Big feast for predators *on that day*
    - But most of prey survive because predators full!
  - But tendency for population explosions/collapses
    - Large number survive one year;
    - Population exceeds land carrying capacity
    - Big death levels too…

Fractals and Structure

- Logistic equation models this in 4 ways:

"Discrete" time since births occur once each year

t=year of births

$$L_{t+1} = (1+a)\cdot L_t - b\cdot L_t^2$$

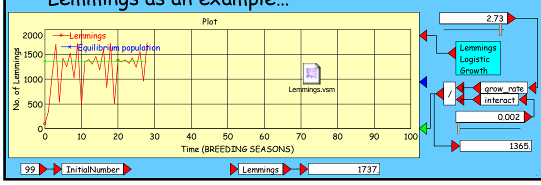
High value for a—lots of children per adult

Negative b times L *squared* captures overcrowding effect on death rate

- Can also be expressed as $x_{t+1} = \alpha x_t(1-x_t)$
- System is realistic "toy" model
- Completely deterministic (no random noise at all); but
  - Behaves "chaotically" for some values of a & b (& α)

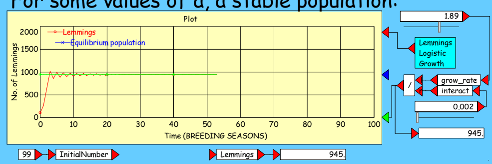
Chaos?

- One of several terms
  - Chaotic
  - Complex
- Used to describe
  - Deterministic systems (maybe with some noise)
  - That are highly unstable & unpredictable
  - Despite existence of underlying structure…
- Lemmings as an example…

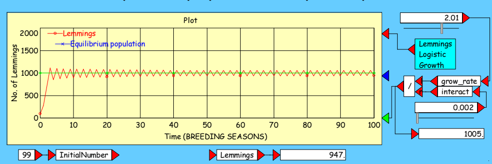


Chaos

- For some values of a, a stable population:

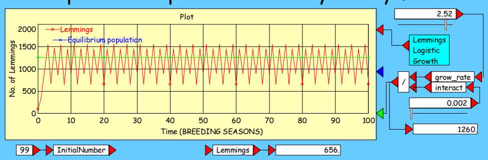


- For a=2, a cyclical population: up one year, down the next

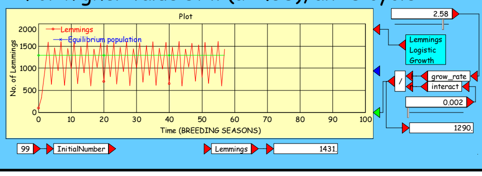


Chaos

- For higher value (a>2.5), a "4 cycle"
  - Population repeats 4 values cyclically forever:

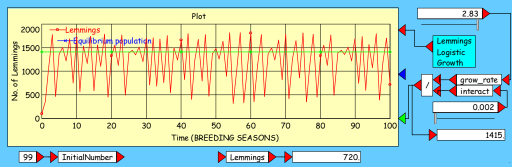


- For higher value still (a>2.58), an "8 cycle"



Chaos

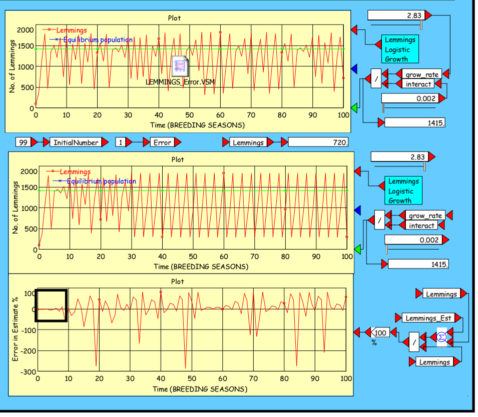
- Ultimately, "chaos"



- Population fluctuates forever—never at equilibrium
- No number ever repeats
- Even though model known precisely, can't predict future
  - Smallest error blown out over time…

Chaos

- Get estimated population wrong by 1%;
- After very few cycles, estimates completely wrong…
- Prediction accurate for under 10 years

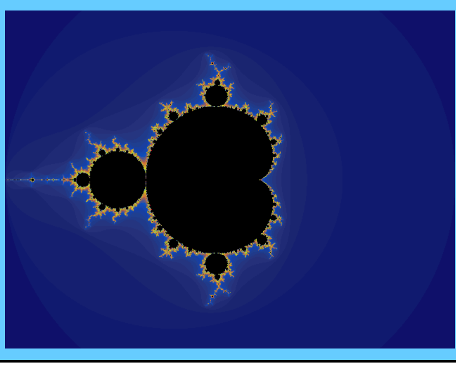


Chaos & Complexity

- Many other instances of chaotic & complex systems
  - Basic features:
    - Current value depends on previous value
      - Unlike random process, or EMH
    - In a highly nonlinear way
      - Subtracting square of number (Logistic)
      - Two variables multiplied together (Lorenz)
    - Patterns generated unpredictable
    - But structure beneath apparent chaos
      - "Self-similarity"
- One of earliest & most beautiful: the Mandelbrot Set

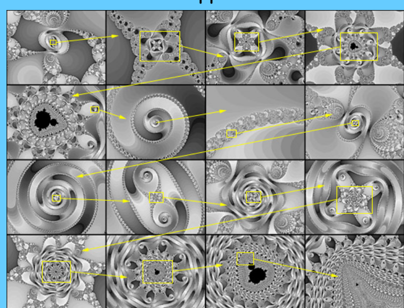
Mandelbrot Set

- A beautiful pattern…



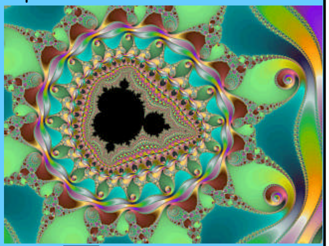
Mandelbrot Set

- With "self-similarity"
  - Zoom in on part
    - The "whole" reappears there!



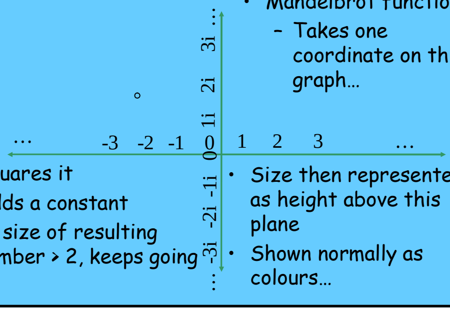
Mandelbrot Set

- Generated by incredibly simple rule:
- Take a number Z
- Square it
- Add a constant
- If the magnitude of the number exceeds 2, keep going
- Otherwise stop
- Just one complication
  - Z & C are "complex numbers": *x+iy* where $i = \sqrt{-1}$
- Complex Numbers fundamental concept in physics
- Essential to understand cyclical systems (eg electricity)
- Represented on x-y plot



Complex Numbers!

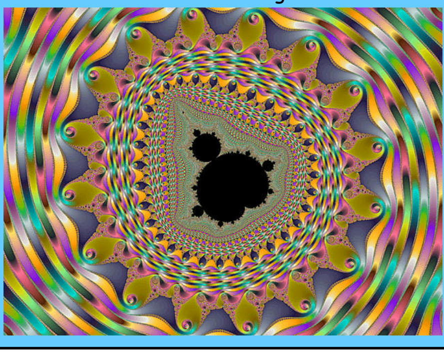
- Real numbers on the horizontal
- "Imaginary" numbers (multiples of square root of minus one) on the vertical:



- Mandelbrot function
  - Takes one coordinate on this graph…
- Squares it
- Adds a constant
- If size of resulting number > 2, keeps going
- Size then represented as height above this plane
- Shown normally as colours…

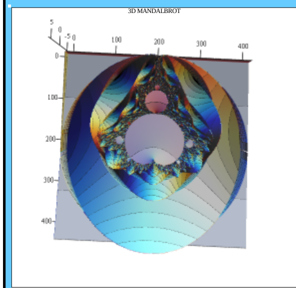
Mandelbrot Set

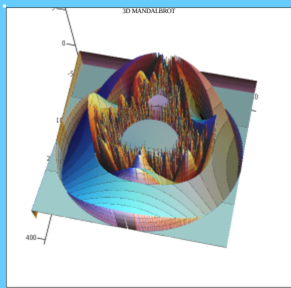
- Black bits are parts where height is zero
- Coloured bits are where height > 0



Mandelbrot Set

- Actual object looks this this…
- Or side on, like this…





- Main relevance of chaos & complexity theory to finance…

Chaos, Complexity & Finance

- Superficially random behaviour can actually have deterministic causes
- If sufficiently strong feedbacks
  - Subtract square of number of lemmings from number of lemming births
  - Two variables times each other in Lorenz
- System can display "chaos"
  - Aperiodic cycles ("booms and busts")
  - Impossible to predict behaviour
    - For more than a few periods ahead
    - Even if you know underlying dynamic precisely!
- Alternative explanation for "it's hard to beat the market"
  - To "because it's rational" view of EMH

Fractal Market Hypothesis (FMH)

- Proposed by Peters (1994)
  - Market is complex & chaotic
  - Market stability occurs when there are many participating investors with different investment horizons.
  - Stability breaks down when all share the same horizon
    - "Rush for the exits" causes market collapse
    - "Stampede" for the rally causes bubble
  - Distribution of returns appears the same across all investment horizons
    - Once adjustment is made for scale of the investment horizon, all investors share the same level of risk.

The "Fractal Markets Hypothesis"

- Peters applies fractal analysis to time series generated by asset markets
  - Dow Jones, S&P 500, interest rate spreads, etc.
  - finds a "fractal" structure
  - intellectually consistent with
    - Inefficient Markets Hypothesis
    - Financial Instability Hypothesis
  - Based upon
    - heterogeneous investors with *different* expectations, *different* time horizons
      - trouble breaks out when all investors suddenly operate on *same* time horizon with *same* expectations

The "Fractal Markets Hypothesis"

- "Take a typical day trader who has an investment horizon of five minutes and is currently long in the market.
  - The average five-minute price change in 1992 was -0.000284 per cent [it was a "bear" market], with a standard deviation of 0.05976 per cent.
- If … a six standard deviation drop occurred for a five minute horizon, or 0.359 per cent, our day trader could be wiped out if the fall continued.
- However, an institutional investor–a pension fund, for example–with a weekly trading horizon, would probably consider that drop a buying opportunity
  - because weekly returns over the past ten years have averaged 0.22 per cent with a standard deviation of 2.37 per cent.

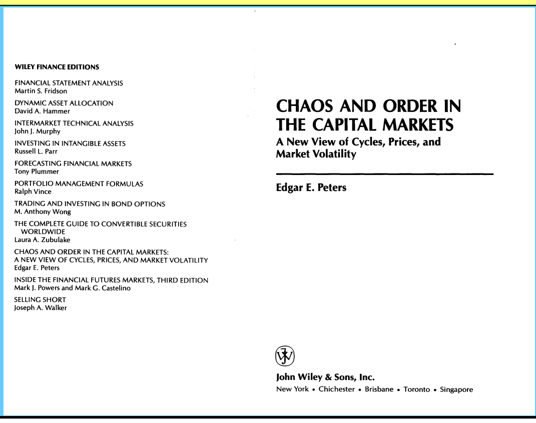
The "Fractal Markets Hypothesis"

- In addition, the technical drop has not changed the outlook of the weekly trader, who looks at either longer technical or fundamental information.
- Thus the day trader's six-sigma [standard deviation] event is a 0.15-sigma event to the weekly trader, or no big deal.
- The weekly trader steps in, buys, and creates liquidity.
- This liquidity in turn stabilises the market." (Peters 1994)

The "Fractal Markets Hypothesis"

- Peters uses Hurst Exponent as another measure of chaos in finance markets
- Didn't have time to complete this part of lecture
- In lieu, next slides extract Chapter 7 of Chaos And Order In The Capital Markets
  - Explains how Hurst Exponent Derived
- Chapter 8 (in Reading Assignment) applies Hurst Exponent to Share Market…
  - Read these next slides before reading Chapter 8
    - Not expected to be able to reproduce Hurst technique
    - But to understand basic idea
    - And how it shows market structure "fractal"
      - Rather than "random"

The "Fractal Markets Hypothesis"



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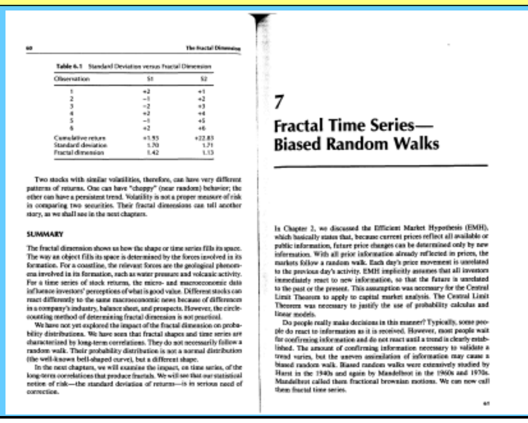

60 The Fractal Dimension

Table 6.1 Standard Deviation versus Fractal Dimension

Observation	S1	S2
1	+2	+1
2	−1	+2
3	−2	+3
4	+2	+4
5	−1	+5
6	+2	+6
Cumulative return	+1.93	+22.83
Standard deviation	1.70	1.71
Fractal dimension	1.42	1.13

Two stocks with similar volatilities, therefore, can have very different patterns of returns. One can have "choppy" (near random) behavior; the other can have a persistent trend. Volatility is not a proper measure of risk in comparing two securities. Their fractal dimensions can tell another story, as we shall see in the next chapters.

SUMMARY

The fractal dimension shows us how the shape or time series fills its space. The way an object fills its space is determined by the forces involved in its formation. For a coastline, the relevant forces are the geological phenomena involved in its formation, such as water pressure and volcanic activity. For a time series of stock returns, the micro- and macroeconomic data influence investors' perceptions of what is good value. Different stocks can react differently to the same macroeconomic news because of differences in a company's industry, balance sheet, and prospects. However, the circle-counting method of determining fractal dimension is not practical.

We have not yet explored the impact of the fractal dimension on probability distributions. We have seen that fractal shapes and time series are characterized by long-term correlations. They do not necessarily follow a random walk. Their probability distribution is not a normal distribution (the well-known bell-shaped curve), but a different shape.

In the next chapters, we will examine the impact, on time series, of the long-term correlations that produce fractals. We will see that our statistical notion of risk—the standard deviation of returns—is in serious need of correction.

7

Fractal Time Series—Biased Random Walks

In Chapter 2, we discussed the Efficient Market Hypothesis (EMH), which basically states that, because current prices reflect all available or public information, future price changes can be determined only by new information. With all prior information already reflected in prices, the markets follow a random walk. Each day's price movement is unrelated to the previous day's activity. EMH implicitly assumes that all investors immediately react to new information, so that the future is unrelated to the past or the present. This assumption was necessary for the Central Limit Theorem to apply to capital market analysis. The Central Limit Theorem was necessary to justify the use of probability calculus and linear models.

Do people really make decisions in this manner? Typically, some people do react to information as it is received. However, most people wait for confirming information and do not react until a trend is clearly established. The amount of confirming information necessary to validate a trend varies, but the uneven assimilation of information may cause a biased random walk. Biased random walks were extensively studied by Hurst in the 1940s and again by Mandelbrot in the 1960s and 1970s. Mandelbrot called them fractional brownian motions. We can now call them fractal time series.

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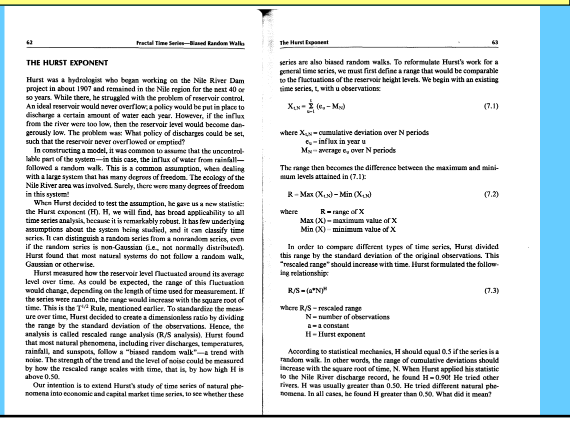

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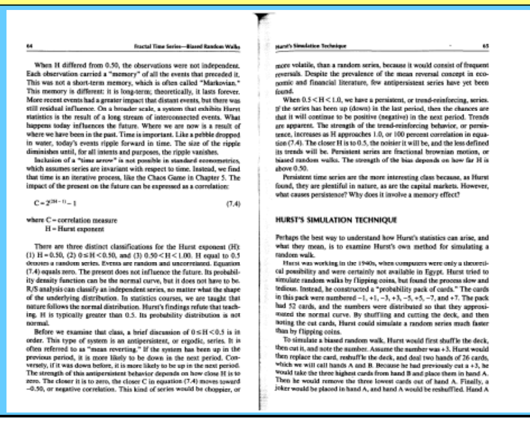

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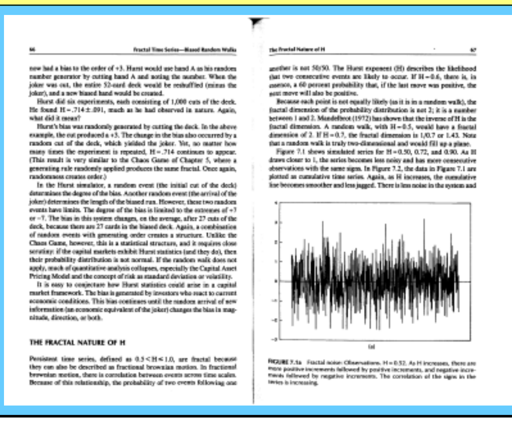

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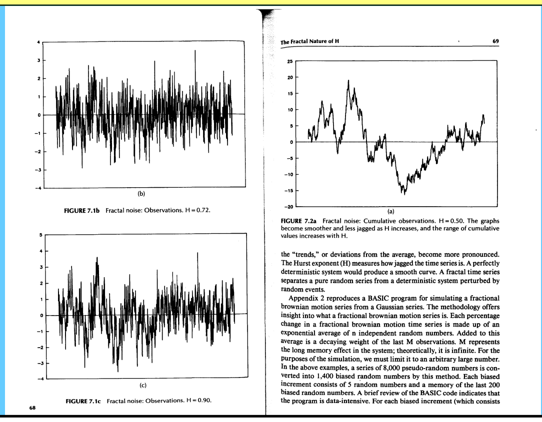

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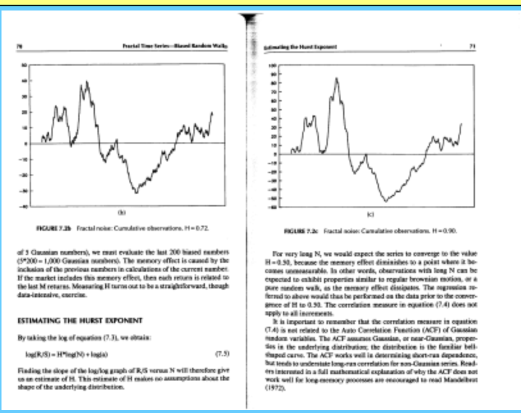

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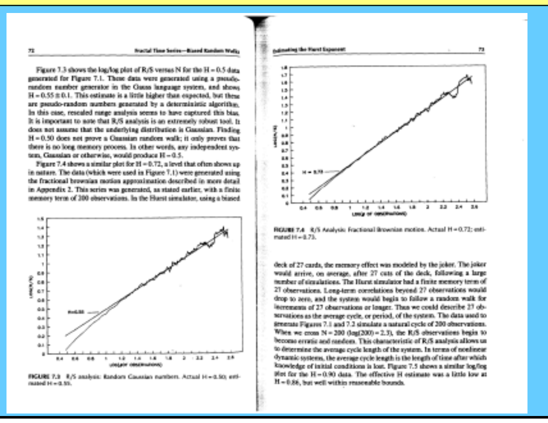

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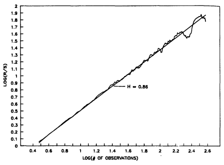

FIGURE 7.5 R/S analysis: fractional Brownian motion. Actual H = 0.90; estimated H = 0.86.

HOW VALID IS THE H ESTIMATE?

Even if a significantly anomalous value of H is found, there may still be a question as to whether the estimate itself is valid. Perhaps there were not enough data, or there may even be a question as to whether R/S analysis works at all. I suggest the following simple test, which is based on a test developed by Scheinkman and LeBaron (1986) for correlation dimension (which we shall study in Chapter 12).

Essentially, an estimate of H that is significantly different from 0.50 has two possible explanations:

1. There is a long memory component in the time series being studied. Each observation is correlated to some degree with the observations that follow.
2. The analysis itself is flawed, and an anomalous value of H does not mean that there is a long memory effect at work.

Perhaps we do not have enough data for a valid test, given that guidelines as to the correct amount of data are somewhat fuzzy. Still, the series being studied is an independent series of random variables, which happens to (1) scale according to a value different from 0.50, or (2) be an independent process with fat tails, as suggested by Cootner (1964).

We can test the validity of our results by randomly scrambling the data so that the order of the observations is completely different from that of the original time series. Because the actual observations are all still there, the frequency distribution of the observations remains unchanged. Now we repeat the calculation of the Hurst exponent on the scrambled data. If the series is truly an independent series, then the Hurst exponent calculation should remain virtually unchanged, because there were no long memory effects, or correlations, between the observations. Therefore, scrambling the data would have no effect on the qualitative aspect of the data.

If there was a long memory effect in place, the order of the data is important. By scrambling the data, we should have destroyed the structure of the system. The H estimate we calculate should be much lower, and

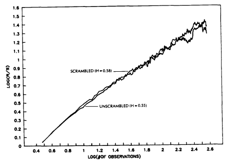

FIGURE 7.6 Scrambling test for R/S analysis: random Gaussian numbers. Unscrambled H = 0.55; scrambled H = 0.58.

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closer to 0.50, even though the frequency distribution of the observations remains unchanged.

I have done such a scrambling test for the simulated series discussed above. First, I scrambled the random series, which had an effective H value of 0.55. Figure 7.6 shows the log/log plot for the scrambled and unscrambled series. There is virtually no qualitative difference between the two. The scrambled series gives H = 0.58 as its estimate. Scrambling actually increased the Hurst exponent, showing that the original series did not truly have a long memory process in place.

Figure 7.7 shows the log/log plot for H = 0.90 of the scrambled and unscrambled series. Here, a qualitative difference appears. The original series gave an H estimate of 0.87. The scrambled series gives H = 0.52. This drop in the value of H shows that the long memory process in the original time series was destroyed by the scrambling process. The scrambled series still has a nonnormal frequency distribution, but the scrambling process determined that the observations were independent. This

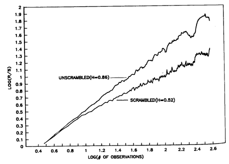

FIGURE 7.7 Scrambling test for R/S analysis: fractional Brownian motion. Unscrambled H = 0.86; scrambled H = 0.52.

proves Mandelbrot's assertion that R/S analysis is robust with respect to the distribution of the underlying series.

R/S ANALYSIS OF THE SUNSPOT CYCLE

Before we analyze the capital markets in the next chapter, it would be useful to apply R/S analysis to a time series of real data from a natural system.

Perhaps the most widely known natural system with a nonperiodic cycle is the sunspot cycle. Sunspot numbers have been recorded since the mid-18th century, when Wolf began a daily routine of examining the sun's face through his telescope and counting the number of black spots on its surface. When he died, the Zurich Observatory continued this practice, as it does to this very day. In one procedure inherited from Wolf, a cluster of closely spaced sunspots is counted as one large spot. Thus, five sunspots yesterday could become one large spot today. Combined with errors common in a

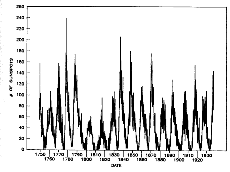

FIGURE 7.8 Wolf's monthly sunspot numbers, January 1749–December 1937.

The "Fractal Markets Hypothesis"

manual procedure, this process lends itself to a certain degree of measurement error. Also, the number of sunspots is a highly asymmetric distribution: it can be as low as zero (which it has been at numerous times), but the maximum number can reach any level. In addition, the sunspot cycle is considered nonperiodic, with an average duration estimated at 11 years.

Sunspots offer a highly appropriate time series for R/S analysis, given their long recorded history. My local library carries an old book by Harlan True Stetson, *Sunspots and Their Effects,* which was published in 1938. It contains a table of monthly sunspot numbers from January 1749 through December 1937. Mandelbrot and Wallis (1969) and Hurst analyzed sunspot data as well. However, it is useful to redo an analysis, incorporating the advances in technology since the last study. Please note that I am *not* making a connection between the sunspot cycle and capital market or economic cycles. I am analyzing sunspots as a cycle in their own right.

Figure 7.8 shows the monthly sunspot numbers as a time series. Note that, although its "cycles" are clearly apparent, the time series is very

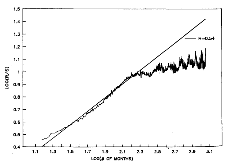

FIGURE 7.9 R/S Analysis: Wolf's monthly sunspot numbers, January 1749–December 1937.

jagged. R/S analysis was applied to the logarithmic first difference in the monthly sunspot numbers.

Figure 7.9 shows the log/log plot of R/S versus time. We can see that periods shorter than 12 to 13 years have a Hurst exponent of 0.55. While not highly anomalous, it shows that sunspots do exhibit persistent behavior. Interestingly, the slope of the log/log plot drops drastically after this point, showing that the long memory effect has dissipated by 10 to 13 years. This is roughly equivalent to the estimated 11-year cycle accepted by scientists.

Figure 7.10 shows the result of the scramble test on the monthly sunspot numbers. The Hurst exponent is now measured at 0.50, and all trace of the memory length has been destroyed by the scrambling process.

From this, we can see that natural systems may have long memories as postulated by the model of fractional brownian motion. However, the memory length is not infinite; it is long and finite. This result is similar to the relationship between natural and mathematical fractals. As we have seen, mathematical fractals scale forever, both infinitely small and

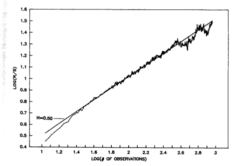

FIGURE 7.10 R/S analysis: scrambled sunspot numbers, 1749–1937.

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large. However, natural fractals stop scaling after a point. Branches of our lungs, for instance, do not become infinitely small. In a similar manner, fractal time series have long, but finite memories. As we study the capital markets and economic time series, we will find similar characteristics. Economic and capital market time series are characterized by long but finite memories. We will also find that the length of these memory cycles varies from market to market, as well as from security to security.

SUMMARY

From R/S analysis, two important items of information can be determined: the Hurst exponent (H) and the average cycle length. The existence of a cycle length has important implications for momentum analysis. A value of H different from 0.5 means that the probability distribution is not normally distributed. If $0.5 < H < 1$, then the series is fractal. Fractal time series behave differently than random walks. We have already discussed persistence and long-term correlations, but there are other differences as well. These differences will be more closely examined in Chapter 9. First, we will do some capital market analysis.

8 R/S Analysis of the Capital Markets

Applying R/S analysis is simple and straightforward, but it requires a fair amount of data and number crunching. In this chapter, we will describe and show the results of applying R/S analysis to various capital markets. In all cases, we find fractal structure and nonperiodic cycles—conclusive evidence that the capital markets are nonlinear systems and that the EMH is questionable. The analysis presented in this chapter is an extension of Peters (1989, 1991b).

METHODOLOGY

When analyzing markets, we use logarithmic returns, defined as follows:

$$S_t = \ln(P_t/P_{t-1}) \qquad (8.1)$$

where S_t = logarithmic return at time t
P_t = price at time t

For R/S analysis, logarithmic returns are more appropriate than the more commonly used percentage change in prices. The range used in R/S analysis is the cumulative deviation from the average, and logarithmic returns sum to cumulative return, while percentage changes do not.

Mandelbrot Set